

# Neutrino masses, mixing, and leptogenesis in an S3 model

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## Abstract

In this work we use previous results on the masses and mixing of neutrinos of an S3 model with three right-handed Majorana neutrinos and three Higgs doublets, to reduce one parameter in the case when two of the right-handed neutrinos are mass degenerate. We derive a new parameterization for the  $V_{PMNS}$  mixing matrix, with a new set of parameters, in the more general case where the right-handed neutrino masses are different. With these results, we calculate leptogenesis and the associated baryogenesis in the model in the two different scenarios. We show that it is possible to have enough leptogenesis to explain the baryonic asymmetry with right-handed neutrino masses above  $10^6$  GeV.

## 1 Introduction

The Standard Model (SM) is extremely successful, nevertheless the discovery of neutrino masses and mixing in neutrino oscillation experiments in 1998 [1], presented evidence that is necessary to go beyond it. Even before this discovery, the amount of free parameters and the hierarchy problem, among others, have prompted attempts to find a more fundamental theory, of which the SM is the low-energy limit [2–4]. Some of the goals of these new models are to understand the large differences in the Yukawa couplings of the different fermions, the hierarchy between the fundamental particles, and the amount of CP violation and the structure of the CKM matrix [5]. A popular way to approach these problems is to build models with Non-Abelian flavor symmetries, often supplemented with extra Higgs doublets. Common symmetries in flavor theories are, among many others, A4, Q6 or S3 [6–11]. The reason is that these models achieve in a natural way the Nearest Neighbour Interaction textures in the fermion mass matrices [12,13]. The S3 extension of the SM with three Higgs doublets (S3-3H) [10,11,14] is a model in which a symmetry on the permutation of three objects is imposed, which in addition to the SM particles has another two Higgs doublets, as well as three right-handed Majorana neutrinos, which are related to the left ones through the seesaw mechanism (type I).

There has been a lot of work done on various S3 models (see for instance [15–22]), some of this work reproduces the CKM and PMNS matrices in agreement with the current experimental data [10,11,23–26], and there have been

also studies of leptogenesis in a soft breaking S3 model [27]. Nevertheless, most of this work has been done in the case where two right-handed neutrinos are degenerate. In this way, it is an interesting question to extend the model and see the possible new results with a generalization, taking into account both degenerate and non-degenerate right-handed neutrino masses. Following the idea of previous work [28], we extend the analysis on the generalization of the S3-3H model.

Another question that the SM fails to explain is the observed baryon asymmetry. It is well known that there are more baryons than antibaryons in the Universe. Nucleosynthesis is a solid and consistent model of the creation of the nuclei in the early Universe, which predicts a baryonic density of,

$$\eta = \frac{\eta_b - \eta_{\bar{b}}}{\eta_\gamma} = \eta = (2.6 - 6.2) \times 10^{-10}. \quad (1)$$

Measurements of the Cosmic Background Radiation [29–31] show a density of

$$\eta = (6.1 \pm 0.3) \times 10^{-10}, \quad (2)$$

in full agreement with the baryon density of the Nucleosynthesis [30,32].

The idea to explain the baryon asymmetry through a dynamical process was proposed by Sakharov in 1967 [33]. The present cosmological observations favour the idea that the matter-antimatter asymmetry of the Universe may be explained in terms of a dynamical generation mechanism, called baryogenesis. Also, it has been realized that a successful model of baryogenesis cannot occur within the Standard Model (SM).

Leptogenesis is a mechanism which generates baryon asymmetry by creating a leptonic asymmetry through  $B + L$  violating electroweak sphaleron transitions [34].

Several things are needed for the occurrence of leptogenesis:

- Heavy right handed neutrinos.
- Majorana type neutrinos.
- Decay of the right handed neutrinos to the left ones.

According to the original proposal of Fukugita and Yanagida [35], this mechanism also satisfies all the Sakharov's conditions [33] in order to produce a net baryon asymmetry (for reviews see for instance [36–38]).

In this paper we explore the possibility of leptogenesis in the S3-3H model, with degenerate and non-degenerate right-handed neutrino masses, and calculate the associated baryogenesis. We first study the case where two of the right-handed neutrino masses are degenerate, and then the more general case where all the right-handed neutrino masses are different. We scan the parameter space to find the leptogenesis and associated baryogenesis dependence on the free parameters of the model. We find that there is a region of parameter space where enough baryogenesis is produced through leptogenesis to explain the baryon asymmetry of the Universe.

The outline of the paper is organized as follows: In section 2, the S3 model is

introduced as well as some of its most important results. In section 3 it is shown how to produce leptogenesis in the S3-3H model, and the resultant baryogenesis is also computed. At the end, in section 4, we conclude summarizing our main results.

## 2 S3-3H model

In the Standard Model analogous fermions in different generations have identical couplings to all gauge bosons of the strong, weak, and electromagnetic interactions [16]. The group S3 consists of the six possible permutations of three objects  $(f_1, f_2, f_3)$ , and is the smallest discrete non-abelian group. It has one 2-dimension irreducible representation (irrep) and two of 1-dimension

$$F_s = (f_1, f_2, f_3), \quad F_{d1} = -f_1 - f_2 + 2f_3, \quad (3)$$

$$f_{d2} = f_2 - f_3. \quad (4)$$

We can associate the particles in the model to doublets or to singlets with the following rules. The direct product of two doublets  $p_D^T = (p_{D1}, p_{D2})$  and  $q_D^T = (q_{D1}, q_{D2})$  may be decomposed into the direct sum of two singlets  $r_s$  and  $r_{s'}$ , and one doublet  $r_D^T$  where

$$r_s = p_{D1}q_{D1} + p_{D2}q_{D2} \quad r_{s'} = p_{D1}q_{D2} - p_{D2}q_{D1} \quad (5)$$

$$r_D^T = (r_{D1}, r_{D2}) = (p_{D1}q_{D2} + p_{D2}q_{D1}, p_{D1}q_{D1} + p_{D2}q_{D2}). \quad (6)$$

Since the Standard Model has only one Higgs  $SU(2)_L$  doublet, which can only be an  $S_3$  singlet, it gives mass to the particles in the  $S_3$  singlet representation. To give mass to the rest of the particles we extend the Higgs sector of the theory, by adding two more Higgs doublets. The quark and Higgs fields are

$$Q^T = (u_L, d_L), u_r, d_r, \quad (7)$$

$$L^t = (\nu_L, e_L), e_R, \nu_R \quad \text{and} \quad H. \quad (8)$$

All of the fields have three species, and we assume that each one forms a reducible representation  $1S \oplus 2$ . The first two generations will be assigned to the doublet S3 irrep, and the third generation to the singlet. This applies to quarks, leptons, Higgs fields, and right-handed neutrinos. The doublets carry capital indices  $I$  and  $J$ , which run from 1 to 2, and the singlets are denoted by  $Q_3, u_{3R}, d_{3R}, L_3, e_{3R}, \nu_{3R}$  and  $H_S$ . The subscript 3 denotes the singlet representation and not the third generation. The most general renormalizable Yukawa interactions of this model are given by [10]

$$L_Y = L_{Y_D} + L_{Y_U} + L_{Y_E} + L_{Y_\nu} \quad (9)$$

where

$$\begin{aligned} L_{Y_D} = & -Y_l^d \overline{Q_I} H_S d_{IR} - Y_3^d \overline{Q_3} H_S d_{3r} \\ & - Y_2^d [\overline{Q_I} \kappa_{IJ} H_l d_{JR} - \overline{Q_I} \eta_{IJ} H_2 d_{JR}] \\ & - Y_4^d \overline{Q_3} H_I d_{IR} - Y_5^d \overline{Q_I} H_I d_{3R} + h.c. \end{aligned} \quad (10)$$

$$\begin{aligned}
L_{Y_U} = & -Y_1^u \overline{Q_I} (i\sigma_2 H_S^* u_{IR}) - Y_3^u \overline{Q_3} (i\sigma_2 H_S^* u_{3R}) \\
& - Y_2^u [\overline{Q_I} \kappa_{IJ} (i\sigma_2 H_1^* u_{JR}) - \overline{Q_I} \eta_{IJ} (i\sigma_2 H_2^* u_{JR})] \\
& - Y_4^u \overline{Q_3} (i\sigma_2 H_I^* u_{IR}) - Y_5^u \overline{Q_I} (i\sigma_2 H_I^* u_{3R}) + h.c.
\end{aligned} \tag{11}$$

$$\begin{aligned}
L_{Y_U} = & -Y_1^u \overline{Q_I} (i\sigma_2 H_S^* u_{IR}) - Y_3^u \overline{Q_3} (i\sigma_2 H_S^* u_{3R}) \\
& - Y_2^u [\overline{Q_I} \kappa_{IJ} (i\sigma_2 H_1^* u_{JR}) - \overline{Q_I} \eta_{IJ} (i\sigma_2 H_2^* u_{JR})] \\
& - Y_4^u \overline{Q_3} (i\sigma_2 H_I^* u_{IR}) - Y_5^u \overline{Q_I} (i\sigma_2 H_I^* u_{3R}) + h.c.
\end{aligned} \tag{12}$$

$$\begin{aligned}
L_{Y_E} = & -Y_1^e \overline{L_I} H_S e_{IR}) - Y_3^e \overline{L_3} H_S e_{3R}) \\
& - Y_2^e [\overline{L_I} \kappa_{IJ} H_1 e_{JR} - \overline{L_I} \eta_{IJ} H_2 e_{JR}] \\
& - Y_4^e \overline{L_3} H_I e_{IR} - Y_5^e \overline{L_I} H_I D_{3R} + h.c.
\end{aligned} \tag{13}$$

$$\begin{aligned}
L_{Y_\nu} = & -Y_1^\nu \overline{L_I} (i\sigma_2 H_S^* \nu_{IR}) - Y_3^\nu \overline{L_3} (i\sigma_2 H_S^* \nu_{3R}) \\
& - Y_2^\nu [\overline{L_I} \kappa_{IJ} (i\sigma_2 H_1^* \nu_{JR}) - \overline{L_I} \eta_{IJ} (i\sigma_2 H_2^* \nu_{JR})] \\
& - Y_4^\nu \overline{L_3} (i\sigma_2 H_I^* \nu_{IR}) - Y_5^\nu \overline{L_I} (i\sigma_2 H_I^* \nu_{3R}) + h.c.,
\end{aligned} \tag{14}$$

with

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{15}$$

Furthermore, we add to the Lagrangian the Majorana mass terms for the right-handed neutrinos

$$L_M = -M_1 \nu_{1R}^T C \nu_{1R} - M_2 \nu_{2R}^T C \nu_{2R} - M_3 \nu_{3R}^T C \nu_{3R}. \tag{16}$$

Due to the presence of three Higgs fields, the Higgs potential  $V_H(H_S, H_D)$  is more complicated than that of the Standard Model [39]. In addition to the  $S3$  symmetry, under certain conditions the Higgs potential exhibits a permutational symmetry  $Z2 : H1 \leftrightarrow H2$ , which is not a subgroup of the flavor group  $S3$  [24,25]. The model has as well an Abelian discrete symmetry that we will use as selection rules for the Yukawa couplings in the leptonic sector. In this paper, we will assume that the vacuum respects the accidental  $Z2$  symmetry of the Higgs potential and that

$$\langle H_1 \rangle = \langle H_2 \rangle. \tag{17}$$

With these assumptions, the Yukawa interactions, eqs. (10)-(14) yield mass matrices, for all fermions in the theory, of the general form

$$M = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}. \tag{18}$$

The Majorana mass for the left handed neutrinos  $\nu_L$  is generated by the see-saw mechanism. The corresponding mass matrix is given by

$$M_\nu = M_{\nu D} \widetilde{M}^{-1} (M_{\nu D})^T, \tag{19}$$

where  $\widetilde{M} = \text{diag}(M_1, M_1, M_3)$ . In principle, all entries in the mass matrices can be complex since there is no restriction coming from the S3 flavor symmetry. The mass matrices are diagonalized by bi-unitary transformations as

$$U_{d(u,e)L}^\dagger M_{d(u,e)} U_{d(u,e)R} = \text{diag}(m_{d(u,e)}, m_{s(u,e)}, m_{b(u,e)}) \quad (20)$$

$$U_\nu^T M_\nu U_\nu = \text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}). \quad (21)$$

The entries in this matrix are complex numbers, so the physical masses are their absolute values. The mixing matrices are, by definition,

$$V_{CKM} = U_{uL}^\dagger U_{dL}, \quad V_{PMNS} = U_{eL}^\dagger U_\nu K. \quad (22)$$

Where K is defined as the matrix that take out the phases of the diagonal mass matrix,

$$\text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) = K^\dagger \text{diag}(|m_{\nu 1}|, |m_{\nu 2}|, |m_{\nu 3}|) K. \quad (23)$$

A further reduction of the number of parameters in the leptonic sector may be achieved by means of an Abelian  $Z_2$  symmetry. A possible set of charge assignments of  $Z_2$ , compatible with the experimental data on masses and mixings in the leptonic sector is given in Table I. The  $Z_2$  assignments forbid the following

-	+
$H_S, \nu_{3R}$	$H_I, L_3, L_I, e_{eR}, e_{IR}, \nu_{IR}$

Table 1:  $Z_2$  assignment in the leptonic sector.

Yukawa couplings

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu. \quad (24)$$

Therefore, the corresponding entries in the mass matrices vanish.

## 2.1 Mass matrix for the charged leptons

Under these assumptions, the mass matrix of the charged leptons takes the form

$$M_e = m_\tau \begin{pmatrix} \tilde{\mu}_2 & \tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_2 & -\tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_4 & \tilde{\mu}_4 & 0 \end{pmatrix}. \quad (25)$$

The unitary matrix  $U_{eL}$  that enters in the definition of the mixing matrix,  $V_{PMNS}$ , is calculated from

$$U_{eL}^\dagger M_e M_e^\dagger U_{eL} = \text{diag}(m_e^2, m_\mu^2, m_\tau^2), \quad (26)$$

where  $m_3, m_\mu$  and  $m_\tau$  are the masses of the charged leptons, and

$$M_e M_e^\tau = m_\tau^2 \begin{pmatrix} 2|\tilde{\mu}_2|^2 & |\tilde{\mu}_5|^2 & 2|\tilde{\mu}_2||\tilde{\mu}_4|e^{-i\delta_e} \\ |\tilde{\mu}_5|^2 & 2|\tilde{\mu}_2|^2 & 0 \\ 2|\tilde{\mu}_2||\tilde{\mu}_4|e^{i\delta_e} & 0 & 2|\tilde{\mu}_4|^2 \end{pmatrix}. \quad (27)$$

Notice that this matrix only has one phase factor. The parameters  $|\tilde{\mu}_2|$ ,  $|\tilde{\mu}_4|$  and  $|\tilde{\mu}_5|$  may readily be expressed in terms of the charged lepton masses. From the invariants of  $M_e M_e^\dagger$ , we get the set of equations [17]

$$\text{Tr}(M_e M_e^\dagger) = m_e^2 + m_\mu^2 + m_\tau^2 = m_\tau^2 [4|\tilde{\mu}_2|^2 + 2(|\tilde{\mu}_4|^2 + |\tilde{\mu}_5|^2)] \quad (28)$$

$$\xi(M_e M_e^\dagger) = m_\tau^2(m_e^2 + m_\mu^2) + m_e^2 m_\mu^2 \quad (29)$$

$$= 4m_\tau^4[|\tilde{\mu}_2|^2 + |\tilde{\mu}_2|^2(|\tilde{\mu}_4|^2 + |\tilde{\mu}_5|^2) + |\tilde{\mu}_4|^2|\tilde{\mu}_5|^2] \quad (30)$$

$$(31)$$

$$\det(M_e M_e^\dagger) = m_e^2 m_\mu^2 m_\tau^2 = 4m_\tau^6 |\tilde{\mu}_2|^2 |\tilde{\mu}_4|^2 |\tilde{\mu}_5|^2, \quad (32)$$

where  $\xi(M_e M_e^\dagger) = \frac{1}{2}[Tr(*M_e M_e^\dagger)]^2 - Tr((M_e M_e^\dagger)^2)$ . Solving these equations for  $|\tilde{\mu}_2|$ ,  $|\tilde{\mu}_4|$  and  $|\tilde{\mu}_5|$ , we obtain

$$|\tilde{\mu}_2|^2 = \frac{1}{2} \frac{m_e^2 + m_\mu^2}{m_\tau^2} - \frac{m_e^2 m_\mu^2}{m_\tau^2(m_e^2 + m_\mu^2)} + \beta. \quad (33)$$

In this expression,  $\beta$  is the smallest solution of the equation

$$\beta^3 - \frac{1}{2}(1 - 2y + 6\frac{x}{y})\beta^2 - \frac{1}{4}(y - y^2 - 4\frac{z}{y} + 7z - 12\frac{z^2}{y^2})\beta - \quad (34)$$

$$\frac{1}{8}yz - \frac{1}{2}\frac{z^2}{y^2} + \frac{3}{4}\frac{z^2}{y} - \frac{z^3}{y^3} = 0 \quad (35)$$

$$(36)$$

where  $y = (m_e^2 + m_\mu^2)/m_\tau$  and  $z = \mu_\mu^2 \mu_e^2 / \mu_\tau^4$ .

An estimation of  $\beta$  at good order of magnitude is obtained from [40]

$$\beta \simeq -\frac{m_\mu^2 m_e^2}{2m_\tau^2(m_\tau^2 - (m_\tau^2 + m_e^2))}. \quad (37)$$

The parameters  $|\tilde{\mu}_4|^2$  and  $|\tilde{\mu}_5|^2$  are in terms of  $|\tilde{\mu}_2|^2$ ,

$$|\mu_{\tilde{4},5}|^2 = \frac{1}{4}(1 - \frac{m_\mu^2 + m_e^2}{m_\tau^2} + 4\frac{m_e^2 m_\mu^2}{m_\tau^2(m_e^2 + m_\mu^2)} - \beta) \quad (38)$$

$$\pm \frac{1}{4}(\sqrt{(1 - \frac{m_\mu^2 + m_e^2}{m_\tau^2} + 4\frac{m_e^2 m_\mu^2}{m_\tau^2(m_e^2 + m_\mu^2)})^2 - \beta^2} - \frac{m_\mu^2 m_e^2}{m_\tau^4} \frac{1}{|\tilde{\mu}_2|^2}) \quad (39)$$

$$(40)$$

Once  $M_e M_e^\dagger$  has been reparametrized in terms of the charged lepton masses, it is straightforward to compute  $U_{eL}$  also as a function of the lepton masses. Here we will write the result to order  $(m_\mu m_e / m_\tau^2)^2$  and  $x^4 = (m_e / m_\tau)^4$

$$M_e \simeq m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \frac{\sqrt{1+x^2-\tilde{m}_\mu^2}}{\sqrt{1+x^2}} \\ \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \frac{\sqrt{1+x^2-\tilde{m}_\mu^2}}{\sqrt{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 \end{pmatrix}. \quad (41)$$

The unitary matrix  $U_{eL}$  that diagonalizes  $M_e M_e^\dagger$  and enters in the definition of the neutrino mixing matrix  $V_{PMNS}$ , equation (22), is

$$U_{eL} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_e} \end{pmatrix} \begin{pmatrix} O_{11} & -O_{12} & O_{13} \\ -O_{21} & O_{22} & O_{23} \\ -O_{31} & -O_{32} & O_{33} \end{pmatrix}, \quad (42)$$

where

$$U_{eL} \simeq \begin{pmatrix} O_{11} & -O_{12} & O_{13} \\ -O_{21} & O_{22} & O_{23} \\ -O_{31} & -O_{32} & O_{33} \end{pmatrix} = \quad (43)$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}}x \frac{(1+2\tilde{m}_\mu^2+x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^{12}+12x^4}} & -\frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6+5\tilde{m}_e^{12}}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}x \frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^{12}+12x^4}} & \frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6+5\tilde{m}_e^{12}}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^{12}+12x^4}} & -x \frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2)}{\sqrt{1-\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6+5\tilde{m}_e^{12}}} & \tilde{m}_e \tilde{m}_\mu \frac{\sqrt{1+x^2}}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix} = \quad (44)$$

and where  $\tilde{m}_\mu = m_\mu/m_\tau$ ,  $\tilde{m}_e = m_e/m_\tau$  and  $x = m_e/m_\mu$ .

## 2.2 The mass matrix of the neutrinos

With the  $Z_2$  selection rule (Table 1), the mass matrix of the Dirac neutrinos takes the form

$$M_{\nu D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix} \quad (45)$$

Then, the mass matrix for the left-handed Majorana neutrinos is obtained from the see-saw mechanism,

$$M_\nu = M_{\nu D} \tilde{M}^{-1} (M_{\nu D})^T = \begin{pmatrix} (\frac{1}{M_1} + \frac{1}{M_2})\mu_2^2 & (\frac{1}{M_1} - \frac{1}{M_2})\mu_2^2 & (\frac{1}{M_1} + \frac{1}{M_2})\mu_2\mu_4 \\ (\frac{1}{M_1} - \frac{1}{M_2})\mu_2^2 & (\frac{1}{M_1} + \frac{1}{M_2})\mu_2^2 & (\frac{1}{M_1} - \frac{1}{M_2})\mu_2\mu_4 \\ (\frac{1}{M_1} + \frac{1}{M_2})\mu_2\mu_4 & (\frac{1}{M_1} - \frac{1}{M_2})\mu_2\mu_4 & \frac{\mu_4^2}{M_2} + \frac{\mu_3^2}{M_3} \end{pmatrix}, \quad (46)$$

where  $M_i$  are the right handed neutrino masses appearing in eq. (16).

The non-Hermitian, complex, symmetric neutrino mass matrix  $M_\nu$  may be brought to a diagonal form by a bi-unitary transformation, as

$$U_\nu^T M_\nu U_\nu = \text{diag}(m_{\nu 1} e^{i\phi_1}, m_{\nu 2} e^{i\phi_2}, m_{\nu 3} e^{i\phi_3}) \quad (47)$$

Where  $U_\nu$  is the matrix that diagonalizes the matrix  $M_\nu$ .

### 2.2.1 Neutrino matrix with degenerate masses.

In the case where  $M_1 = M_2$  the mass matrix is reduced to [14]

$$M_\nu = M_{\nu D} \tilde{M}^{-1} (M_{\nu D})^T = \begin{pmatrix} (\frac{1}{M_1} + \frac{1}{M_1})\mu_2^2 & 0 & (\frac{1}{M_1} + \frac{1}{M_1})\mu_2\mu_4 \\ 0 & (\frac{1}{M_1} + \frac{1}{M_1})\mu_2^2 & 0 \\ (\frac{1}{M_1} + \frac{1}{M_1})\mu_2\mu_4 & 0 & \frac{\mu_4^2}{M_1} + \frac{\mu_3^2}{M_3} \end{pmatrix}. \quad (48)$$

With this texture is easy to calculate the  $U_\nu$  matrix that diagonalizes  $M_\nu^\dagger M_\nu$ ,

$$M_\nu^\dagger M_\nu = \begin{pmatrix} |A|^2 + |B|^2 & 0 & A^* B + B^* D \\ 0 & |A|^2 & 0 \\ AB^* + BD^* & 0 & |B|^2 + |D|^2 \end{pmatrix} \quad (49)$$

with  $A = \mu_2^2/M_1$ ,  $B = 2\mu_2\mu_4/M_1$  and  $D = 2\mu_1\mu_2/M_1 + \mu_3^2/M_2$ , this matrix is diagonalized by

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_\nu} \end{pmatrix} \begin{pmatrix} \cos \eta & \sin \eta & 0 \\ 0 & 0 & 1 \\ -\sin \eta & \cos \eta & 0 \end{pmatrix}. \quad (50)$$

If we require that the defining equation (47) be satisfied as an identity, we get the following set of equations:

$$2(\mu_2^\nu)^2/M_1 = m_{\nu 3}, \quad (51)$$

$$2(\mu_2^\nu)^2/M_1 = m_{\nu 1} \cos^2 \eta + m_{\nu 2} \sin^2 \eta, \quad (52)$$

$$2(\mu_2^\nu)(\mu_4^\nu)/M_1 = \sin \eta \cos \eta (m_{\nu 2} - m_{\nu 1}) e^{i\delta_n u}, \quad (53)$$

$$2(\mu_2^\nu)^2/M_1 = m_{\nu 3}, \quad (54)$$

$$2(\mu_2^\nu)(\mu_4^\nu)/M_1 = \sin \eta \cos \eta (m_{\nu 2} - m_{\nu 1}) e^{i\delta_n u}, \quad (55)$$

$$2(\mu_4^\nu)^2/M_1 + 2(\mu_3^\nu)^2/M_3 = (m_{\nu 1} \sin^2 \eta + m_{\nu 2} \cos^2 \eta) e^{-2i\delta_n u}. \quad (56)$$

$$(57)$$

Solving these equations for  $\sin \eta$  and  $\cos \eta$ , we find

$$\sin^2 \eta = \frac{m_{\nu 3} - m_{\nu 1}}{m_{\nu 2} - m_{\nu 1}} \quad \cos^2 \eta = \frac{m_{\nu 2} - m_{\nu 3}}{m_{\nu 2} - m_{\nu 1}}. \quad (58)$$

The unitarity of  $U_\nu$  constrains  $\sin \eta$  to be real and thus  $|\sin \eta| \leq 1$ , this condition fixes the phases  $\phi_1$  and  $\phi_2$  as

$$|m_{\nu 1}| \sin \phi_1 = |m_{\nu 2}| \sin \phi_2 = |m_{\nu 3}| \sin \phi_3. \quad (59)$$

The real phase  $\delta_\nu$  appearing in eq. (50) is not constrained by the unitarity of  $U_\nu$ . Therefore the  $U_\nu$  matrix is,

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_n u} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{m_{\nu 2} - m_{\nu 3}}{m_{\nu 2} - m_{\nu 1}}} & \sqrt{\frac{m_{\nu 3} - m_{\nu 1}}{m_{\nu 2} - m_{\nu 1}}} & 0 \\ 0 & 0 & 1 \\ -\sqrt{\frac{m_{\nu 3} - m_{\nu 1}}{m_{\nu 2} - m_{\nu 1}}} & \sqrt{\frac{m_{\nu 2} - m_{\nu 3}}{m_{\nu 2} - m_{\nu 1}}} & 0 \end{pmatrix}. \quad (60)$$

Now, the mass matrix of the Majorana neutrinos,  $M_\nu$ , may be written in terms of the neutrino masses; from (48) and (55,56,57), we get

$$M_\nu = \begin{pmatrix} m_{\nu 3} & 0 & \sqrt{(m_{\nu 3} - m_{\nu 1})(m_{\nu 2} - m_{\nu 3})} e^{-i\delta_\nu} \\ 0 & m_{\nu 3} & 0 \\ \sqrt{(m_{\nu 3} - m_{\nu 1})(m_{\nu 2} - m_{\nu 3})} e^{-i\delta_\nu} & 0 & (m_{\nu 1} + m_{\nu 2} - m_{\nu 3}) e^{-2\delta_\nu} \end{pmatrix} \quad (61)$$

The only free parameters in these matrices, other than the neutrino masses, are the phase  $\phi_\nu$ , implicit in  $m_{\nu 1}$ ,  $m_{\nu 2}$  and  $m_{\nu 3}$ , and the Dirac phase  $\delta_\nu$ .

Therefore, the theoretical mixing matrix  $V_{PMNS}$ , is given by

$$V_{PMNS}^{th} = \begin{pmatrix} O_{11} \cos \eta + O_{31} \sin \eta e^{i\delta} & O_{11} \sin \eta - O_{31} \cos \eta e^{i\delta} & -O_{21} \\ -O_{12} \cos \eta + O_{32} \sin \eta e^{i\delta} & -O_{12} \sin \eta - O_{32} \cos \eta e^{i\delta} & O_{22} \\ O_{13} \cos \eta - O_{33} \sin \eta e^{i\delta} & O_{13} \sin \eta + O_{33} \cos \eta e^{i\delta} & O_{23} \end{pmatrix} \times K. \quad (62)$$



To obtain the expressions for the mixing angles we need to match the theoretical and PDG expressions for the  $V_{PMNS}$  matrix

$$|V_{PMNS}^{th}| = |V_{PMNS}^{PDG}| \quad (63)$$

meaning  $|V_{ij}^{th}| = |V_{ij}^{PDG}|$ . The standard parametrization of the Particle Data Group is

$$V_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}. \quad (64)$$

We can straightforwardly read the equation for the mixing angles with

$$|\sin\theta_{13}| = |O_{21}| \simeq \frac{1}{\sqrt{2}}x \frac{1 + 4x^2 - \tilde{m}_\mu^4}{\sqrt{1 + \tilde{m}_\mu^2 + 5x^2 - \tilde{m}_\mu^4}}, \quad (65)$$

$$|\sin\theta_{23}| = \frac{|O_{22}|}{\sqrt{1 - O_{21}^2}} \simeq \frac{1}{\sqrt{2}} \frac{1 - 2\tilde{m}_\mu^2 + \tilde{m}_\mu^4}{\sqrt{1 - 4\tilde{m}_\mu^2 + x^2 + 6\tilde{m}_\mu^4}}, \quad (66)$$

and

$$\tan\theta_{12} = \frac{O_{11}\sin\eta - O_{31}\cos\eta}{O_{31}\sin\eta + O_{11}\cos\eta} \quad (67)$$

$$\simeq -\sqrt{\frac{m_{\nu 2} - m_{\nu 3}}{m_{\nu 3} - m_{\nu 1}}} \times \left( \frac{\sqrt{1 + 2x^2 - \tilde{m}_\mu^2(1 + \tilde{m}_\mu^2 + x^2)} - \frac{1}{\sqrt{2}}x(1 + 2\tilde{m}_\mu^2 + 4x^2)\sqrt{\frac{m_{\nu 3} - m_{\nu 1}}{m_{\nu 2} - m_{\nu 3}}}}{\sqrt{1 + 2x^2 - \tilde{m}_\mu^2(1 + \tilde{m}_\mu^2 + x^2)} + \frac{1}{\sqrt{2}}x(1 + 2\tilde{m}_\mu^2 + 4x^2)\sqrt{\frac{m_{\nu 2} - m_{\nu 3}}{m_{\nu 3} - m_{\nu 1}}}} \right). \quad (68)$$

We can express  $\tan\theta_{12}$  in terms of the differences of the square of the masses as

$$\tan^2\theta_{12} = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu 3}|^2 \cos^2\phi_{nu})^{1/2} - |m_{\nu 3}||\cos\phi_{nu}|}{(\Delta m_{13}^2 + |m_{\nu 3}|^2 \cos^2\phi_{nu})^{1/2} + |m_{\nu 3}||\cos\phi_{nu}|} \quad (69)$$

where  $\Delta m_{ij}^2 = m_{\nu i}^2 - m_{\nu j}^2$ .

We can use the experimental values of the masses of the charged leptons and the differences of the square of the masses to fit the mixing angles,

$$(\sin^2\theta_{13})^{th} = 1.1 \times 10^{-5}, \quad (\sin^2\theta_{13})^{xp} = 2.19_{-0.12}^{+0.12} \times 10^{-2}, \quad (70)$$

and

$$(\sin^2\theta_{23})^{th} = .499, \quad (\sin^2\theta_{23})^{xp} = .5_{-0.05}^{+0.05}. \quad (71)$$

From expression (69), we may readily derive expressions for the neutrino masses in terms of  $\tan\theta_{12}$ ,  $\phi_\nu$  and the differences of the squared masses,

$$|m_3| = \frac{\sqrt{\Delta m_{13}^2}}{2 \tan\theta_{12} \cos\phi_\nu} \frac{1 - \tan^4\theta_{12} + r^2}{(1 + \tan^2\theta_{12})(1 + \tan^2\theta_{12} + r^2)}, \quad (72)$$

$$|m_1| = \sqrt{|m_{\nu 3}|^2 + \Delta m_{13}^2}, \quad (73)$$

$$|m_2| = \sqrt{|m_{\nu 3}|^2 + \Delta m_{13}^2(1 + r^2)}, \quad (74)$$

here  $r^2 = \Delta m_{12}^2 / \Delta m_{13}^2 \approx 3 \times 10^{-2}$ . This implies an inverted neutrino mass spectrum  $|m_{\nu 3}| < |m_{\nu 1}| < |m_{\nu 2}|$ . As  $r^2 \ll 1$ , the sum of the neutrino masses is

$$\sum_{i=1}^3 |m_{\nu i}| \approx \frac{\Delta m_{13}^2}{2 \cos \phi_\nu \tan \theta_{12}} (1 + 2 \sqrt{1 + 2 \tan^2 \theta_{12} (2 \cos^2 \phi_\nu - 1) + \tan^4 \theta_{12} - \tan^2 \theta_{12}}). \quad (75)$$

The most restrictive cosmological upper bound [41] for this sum is

$$\sum |m_{\nu}| \leq 0.23 \text{ eV}. \quad (76)$$

This upper bound and the experimentally determined values of  $\tan \theta_{12}$  and  $\Delta m_{i,j}^2$ , give a lower bound for

$$\cos \phi_\nu \geq 0.55 \quad (77)$$

or  $0 \leq \phi_\nu \leq 57^\circ$ . We can use again equation (69) to set the best value of  $\phi$ , we find that with  $\phi = 50^\circ$  we get,

$$\tan \theta_{12} = 0.665288 \quad (78)$$

Hence, setting  $\phi_\nu = 50^\circ$  in our formula, we find

$$m_{\nu 1} = 0.052 \text{ eV}, \quad m_{\nu 2} = 0.053 \text{ eV}, \quad m_{\nu 3} = 0.019 \text{ eV}. \quad (79)$$

The computed sum of the neutrino masses is

$$\left( \sum_{i=1}^3 |m_{\nu i}| \right)^{th} = 0.168508 \text{ eV}, \quad (80)$$

below the cosmological upper bound given in eq. (76), as expected. The above value of  $\phi$  is in agreement with the requirements for leptogenesis, as we will show in section 3. One of the successes of the S3-3H model has been to predict an angle  $\theta_{13}$  different from zero, as well a very accurate angles  $\theta_{12}$  and  $\theta_{23}$ . Nevertheless new experimental results have shown that the angle  $\theta_{13}$  is greater than the model predicts with degenerate right-handed neutrino masses. This is the major reason to extend the model further, to the non-degenerate case [28], and where the angles fit the experimental value.

### 2.2.2 The mass matrix of the neutrinos without degeneration

In a more extensive analysis than [28], we continue to study the case where the RHN masses are non-degenerate. The effective neutrino mass matrix  $m_\nu$  is,

$$M_\nu = M_{\nu D} \tilde{M}^{-1} (M_{\nu D})^T = \begin{pmatrix} \left( \frac{1}{\bar{M}_1} + \frac{1}{\bar{M}_2} \right) \mu_2^2 & \left( \frac{1}{\bar{M}_1} - \frac{1}{\bar{M}_2} \right) \mu_2^2 & \left( \frac{1}{\bar{M}_1} + \frac{1}{\bar{M}_2} \right) \mu_2 \mu_4 \\ \left( \frac{1}{\bar{M}_1} - \frac{1}{\bar{M}_2} \right) \mu_2^2 & \left( \frac{1}{\bar{M}_1} + \frac{1}{\bar{M}_2} \right) \mu_2^2 & \left( \frac{1}{\bar{M}_1} - \frac{1}{\bar{M}_2} \right) \mu_2 \mu_4 \\ \left( \frac{1}{\bar{M}_1} + \frac{1}{\bar{M}_2} \right) \mu_2 \mu_4 & \left( \frac{1}{\bar{M}_1} - \frac{1}{\bar{M}_2} \right) \mu_2 \mu_4 & \frac{\mu_4^2}{\bar{M}_2} + \frac{\mu_3^2}{\bar{M}_3} \end{pmatrix}. \quad (81)$$

We are going to assume that the phases of the  $\mu_3$  and  $\mu_4$  terms are aligned, therefore we can write  $M_\nu$  in polar form  $M_\nu = P \bar{M}_\nu P$ , with  $\bar{M}_\nu$ , real and

$P = \text{diag}(e^{-i\theta_{\mu^2}}, e^{-i\theta_{\mu^2}}, e^{i(\frac{\epsilon_a}{2} - \theta_{\mu^4})})$  In this way  $\widetilde{M}_\nu$  can be expressed in terms of a matrix with two texture zeros class I as:

$$\widetilde{M}_\nu = \mu_0 \mathbb{I}_{3 \times 3} + M'_\nu \quad (82)$$

with  $\mu_0 = \frac{2|\mu_2|^2}{M_1}$ .

Therefore, the  $U_\nu$  matrix is  $P^\dagger U_1$  where  $U_1$  is the matrix that diagonalizes  $M'_\nu$ .

We can take a rotation  $u_{\pi/4}$ ,

$$u_{\pi/4} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}, \quad (83)$$

to the  $M'_\nu$  matrix,

$$u_{\pi/4}^T M'_\nu u_{\pi/4} = na \begin{pmatrix} 0 & 0 & 1 \\ 0 & \sqrt{2}\psi(\psi_2 - 1) & \psi_2 \\ 1 & \psi_2 & \mu_c \end{pmatrix} \quad (84)$$

with  $na = M_2/(\sqrt{2}|\mu_2||\mu_4|)$ ,  $\psi = \frac{\sqrt{2}|\mu_2|}{|\mu_4|}$ ,  $\psi_2 = \frac{M_2}{M_1}$  and  $\mu_c = \frac{\mu_4^2}{M_1} + \frac{\mu_3^2}{M_3} - \mu_0$ . The Matrix that diagonalizes  $u_{\pi/4}^T M'_\nu u_{\pi/4}$  is

$$U_2 = \begin{pmatrix} \psi_2 n_1 & \psi_2 n_2 & \psi_2 n_3 \\ -(1 + \mu_c \lambda_1 - \lambda_1^2) n_1 & -(1 + \mu_c \lambda_2 - \lambda_2^2) n_2 & -(1 + \mu_c \lambda_3 - \lambda_3^2) n_3 \\ \psi_2 \lambda_1 n_1 & \psi_2 \lambda_2 n_2 & \psi_2 \lambda_3 n_3 \end{pmatrix}. \quad (85)$$

where  $n_i$  is a normalization factor and  $\lambda_i$  is the  $i$ -th eigenvalue of  $M'_\nu$ . With  $U_1 = u_{\pi/4} U_2$ , therefore

$$U_\nu = \begin{pmatrix} O'_{11} & O'_{12} & O'_{13} \\ O'_{21} & O'_{22} & O'_{23} \\ O'_{31} & O'_{32} & O'_{33} \end{pmatrix}, \quad (86)$$

$$\begin{pmatrix} n_1(\psi_2 + f_1) & n_2(\psi_2 + f_2) & n_3(\psi_2 + f_3) \\ n_1(-\psi_2 + f_1) & n_2(-\psi_2 + f_2) & n_3(-\psi_2 + f_3) \\ \psi_2 n_1 \lambda_1 & \psi_2 n_2 \lambda_2 & \psi_2 n_3 \lambda_3 \end{pmatrix}, \quad (87)$$

where  $f_i = (-1 - \mu_c \lambda_i + \lambda_i^2)$ . In the same way as in the degenerate scenario we have

$$|V_{PMNS}^{th}| = |V_{PMNS}^{PDG}|, \quad (88)$$

in terms of the mixing angles with

$$s_{13} = O'_{13}, \quad s_{23} = \frac{O'_{23}}{\sqrt{1 - O'^2_{13}}}, \quad s_{12} = \frac{O'_{12}}{\sqrt{1 - O'^2_{13}}}. \quad (89)$$

In the non-degenerate scenario we have three free parameters ( $\psi$ ,  $\psi_2$ ,  $\mu_c$ ) for the neutrino matrix. In this model the PMNS matrix may be obtained numerically. We have used the following values for the masses given in [42]

$$m_e = 0.5109989461 \pm 0.0000000031 \text{ MeV}, \quad (90)$$

$$m_\mu = 105.6583745 \pm 0.0000024 \text{ MeV}, \quad (91)$$

$$m_\tau = 1776.86 \pm 0.12 \text{ MeV}. \quad (92)$$

$$(93)$$

In order to obtain the numerical values for the three free parameters we perform a  $\chi^2$  analysis on the parameter space to find their best fit points

$$\chi^2 = \frac{(\sin(\theta_{12})^2 - \sin(\theta_{12})^2)^2}{\sigma_{\sin \theta_{12}^2}^2} + \frac{(\sin(\theta_{12})^2 - \sin(\theta_{12})^2)^2}{\sigma_{\sin \theta_{12}^2}^2} + \frac{(\sin(\theta_{12})^2 - \sin(\theta_{12})^2)^2}{\sigma_{\sin \theta_{12}^2}^2} \quad (94)$$

where we have taken the following experimental values for the  $V_{PMNS}$  elements [42]

$$\sin^2_{\theta_{12}} = 0.304 \pm 0.014, \quad \sin^2_{\theta_{23}} = 0.50 \pm 0.05, \quad \sin^2_{\theta_{13}} = (2.19 \pm 0.12) \times 10^{-2}. \quad (95)$$

The best values for the free parameters are thus found to be

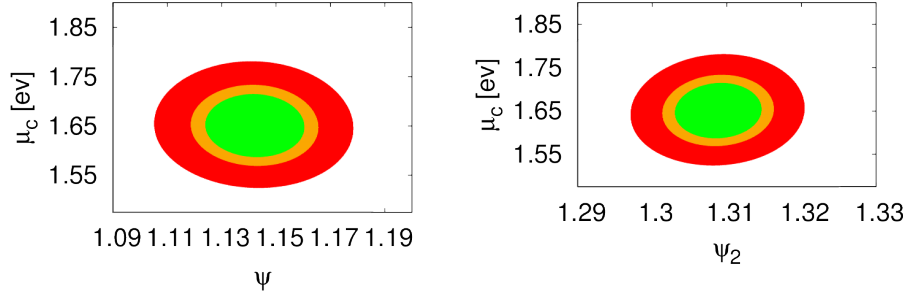
$$\psi_2 = 1.1431, \quad \psi = 1.3091, \quad \mu_c = 1.6502 \text{ eV}, \quad (96)$$

at one sigma C.L. with  $\chi^2 = 3.74 \times 10^{-15}$  as the minimal value. These correspond to the following mixing angles

$$\begin{aligned} \sin(\theta_{12})^2 &= 0.3039, & \sin(\theta_{23})^2 &= 0.4999, \\ \sin(\theta_{13})^2 &= 0.0218. \end{aligned} \quad (97)$$

Fig 1 shows the values for  $(\psi, \psi_2, \mu_c)$  resulting from the  $\chi^2$  analysis. This is part of a more general analysis on the neutrino sector of this model [43].

### Parameter space



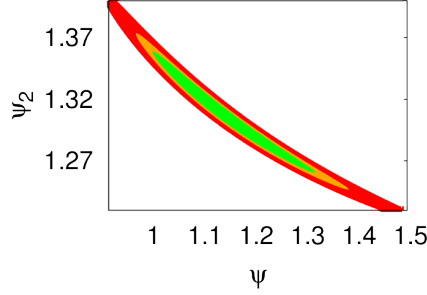


Figure 1: Plot of the free parameter space for the non-degenerate case, where ‘green’ is one, ‘orange’ two, and ‘red’ is three  $\sigma$  agreement with the experimental value.

### 3 Leptogenesis in an S3-3H model

The Yukawa couplings of the neutrinos allow the decay of the right-handed neutrinos into the left-handed ones.

$$-Y_1^\nu \bar{L}_I (i\sigma_2 H_S^* \nu_{IR}). \quad (98)$$

As shown in [38, 44], the asymmetry is defined to be

$$\epsilon_1 = \frac{\sum_\alpha \Gamma(N_1 \rightarrow \ell_\alpha H) - \Gamma(N_1 \rightarrow \bar{\ell}_\alpha \bar{H})}{\sum_\alpha \Gamma(N_1 \rightarrow \ell_\alpha H) + \Gamma(N_1 \rightarrow \bar{\ell}_\alpha \bar{H})}. \quad (99)$$

where  $\Gamma$  is the decay rate, and  $N_1$  is the decaying right-handed neutrino. The possible decays up to tree level are shown in 2,

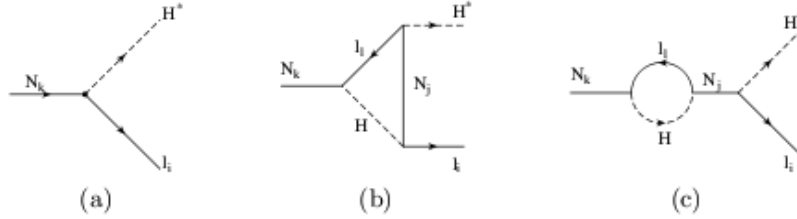


Figure 2: Feynman diagrams up to second order. a) Tree level b) Second Order c) Self Energy diagram

The asymmetry generated by these decays are,

$$\epsilon \simeq -\frac{3}{8\pi} \frac{1}{(h_\nu h_\nu^\dagger)} \sum_{i=2,3} \text{Im}\{(h_\nu h_\nu^\dagger)_{1i}^2\} [f(\frac{M_i^2}{M_1^2}) + g(\frac{M_i^2}{M_1^2})]. \quad (100)$$

and the self interactions are

$$f(x) = \sqrt{x} [1 - (1+x) \ln(\frac{1+x}{x})], \quad (101)$$

$$g(x) = \frac{\sqrt{x}}{1-x} . \quad (102)$$

This function depends strongly on the hierarchy of the light neutrino masses. It can lead to a strong enhancement of the CP asymmetries if the masses  $M_2$  and  $M_3$  are nearly degenerate.

The relation between the lepton and baryon asymmetry is given through the sphaleron process [34]

$$Y_B = aY_{B-L} = \frac{a}{a-1}Y_L , \quad (103)$$

where  $a$  is  $a = (8N_f + 4N_H)/(22N_f + 13N_H)$ ,  $N_f$  is the number of families and  $N_H$  is the number of Higgs doublets.

We can express the lepton asymmetry in terms of the CP asymmetry

$$Y_L = \frac{n_L - n_{\bar{L}}}{s} = \kappa \frac{\epsilon_i}{g^*} , \quad (104)$$

where  $g$  is 110, the number of relativistic degrees of freedom,  $\kappa$  is obtained from solving the Boltzmann equations, and it can be reparametrized in terms of  $K$  defined as the ratio of  $\Gamma_1$ , the tree-level decay width of  $N_1$  to  $H$  the Hubble parameter at temperature  $M_1$ , where  $k = \Gamma_1/H < 1$  describes a process out of thermal equilibrium and  $\kappa < 1$  describes the washout effect [45, 46]:

$$\kappa \approx \frac{0.3}{K(\ln(K))^{0.6}} \quad \text{for } 10 < K < 10^6 , \quad (105)$$

$$\kappa \approx \frac{1}{2\sqrt{K^2 + 9}} \quad \text{for } 0 < K < 10 . \quad (106)$$

The decay width of  $N_1$  by the Yukawa interaction at tree level and Hubble parameter in terms of the temperature  $T$  and the Planck scale  $M_{pl}$  are  $\Gamma_1 = (m_D^\dagger m_D)_{11} M_1 / (8\pi v^2)$  and  $H = 1.66 g^{*1/2} T^2 / M_{pl}$  respectively. At temperature  $T = M_1$  the ratio  $K$  is

$$K = \frac{M_{pl}}{1.66\sqrt{g^*}(8\pi v^2)} \frac{(m_D^\dagger m_D)_{11}}{M_1} . \quad (107)$$

### 3.1 Baryon asymmetry in the degenerate scheme

Putting all the above ingredients together, the asymmetry for the S3-3H model is

$$\epsilon = \frac{\text{Im}[e^{2i\delta^*} M_2 m_3 \frac{\sqrt{M_2(m_2 - m_3)(m_3 - m_1)}}{\sqrt{m_3}}](f[\frac{M_3^2}{M_1^2}] + g[\frac{M_3^2}{M_1^2}])}{8\pi|M_2 m_3|} . \quad (108)$$

The value of the baryon asymmetry has a dependence on  $\phi$  and the masses of the neutrinos  $|m_1|, M_1, |m_2|, M_2, |m_3|, M_3$ , where the masses of the right-handed neutrinos are considered real. We can calculate the dependence of the baryon asymmetry on the phase  $\delta$ . As can be seen from eq. (108) the asymmetry is a periodic function of  $\delta$ , where the masses give the scale of the baryon asymmetry.

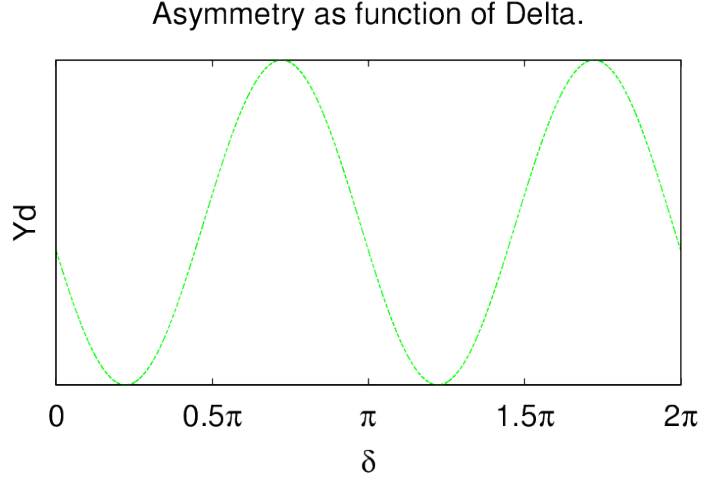


Figure 3: Baryon asymmetry dependance on  $\delta$ .

The maximum value for the baryon asymmetry is on  $\delta = 3/4\pi$ . As figure 3 shows, the leptogenesis is crucially dependent on the phase. The value of the baryon asymmetry is determined by the masses of the light neutrinos and the ratios of the right-handed neutrino masses. The see-saw mechanism relates the masses of the right-handed neutrinos to the light ones, making the right-handed neutrino masses bigger than  $10^{12} \text{ GeV}$ , in order to be in agreement with the experimental data.

We calculate the asymmetry generated in the best case scenario where  $\delta = 3/4\pi$ . In this case we can see from fig. 4 that the masses of the right-handed neutrinos could be of order of  $10^7 \text{ GeV}$  to produce leptogenesis. The graph also shows the region of resonant leptogenesis  $M_1 - M_3 \simeq \frac{1}{2}\Gamma_{N_{1,3}}$ , where the asymmetry increases above the one observed in the Universe, lowering even more the possible mass of the right-handed neutrinos or the  $\delta$  phase.

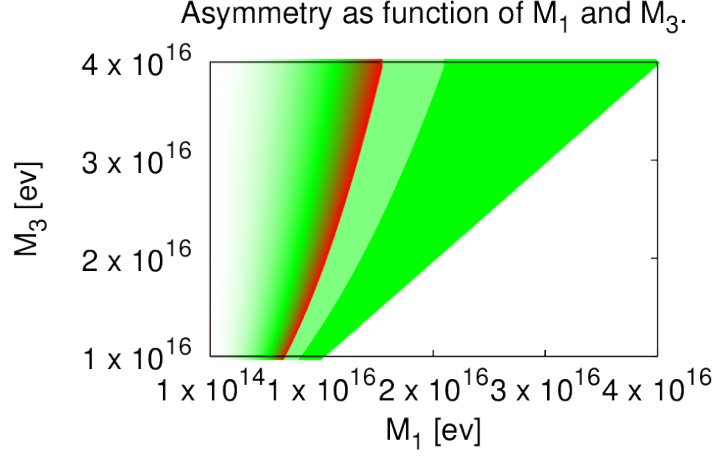


Figure 4: Baryon asymmetry dependence on  $M_1, M_3$ , more asymmetry corresponds to darker shades of green. The red region corresponds to an excess of asymmetry, with the maximum value of  $\delta$ .

### 3.2 Non-degenerate scenario

The value of the baryon asymmetry in the non-degenerate case has dependence on the angles of the  $M_d$  matrix  $\mu_{2a}, \mu_{3a}$  and the real masses of the neutrinos  $M_1, M_2, M_3$ , where  $\mu_{ia}$  is the angle of  $\mu_i = |\mu_i|e^{i\mu_{ia}}$ . We can calculate the dependence of the baryon asymmetry on the phases  $\mu_{2a}, \mu_{3a}$ . As in the degenerate case the magnitude or scale of the baryon asymmetry is given by the masses.

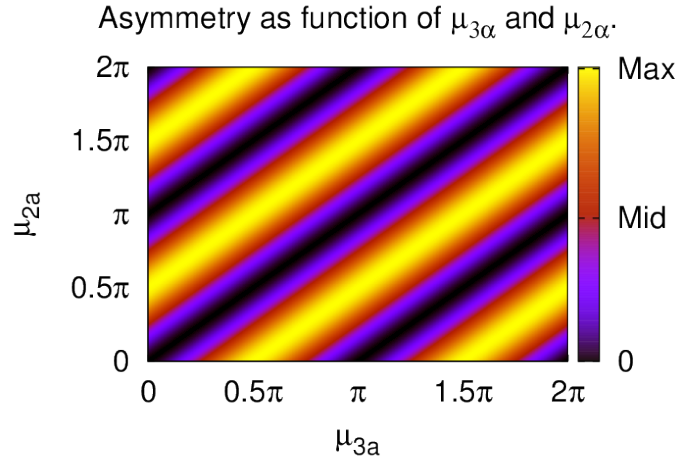


Figure 5: Baryon asymmetry dependance on the phases  $\mu_{2a}, \mu_{3a}$



The maximum of the asymmetry is achieved in all the lines where  $\mu_{2a} - \mu_{3a} = \pi/2 + n\pi$ , where  $n$  is any integer, as can be seen from fig. 5. Again, this is independent of the masses of the neutrinos, the masses only fix the scale of the asymmetry. Taking the best values of the angles we can see that the scale of the masses of the right-handed neutrinos can be lower and that the region of resonant leptogenesis is wider. Therefore, this gives a wider region in parameter space fulfilling the baryon asymmetry explanation. In fig. 6 we show the baryon asymmetry dependence on the  $M_1$  and  $M_3$  masses. The darker shades of green correspond to more asymmetry, whereas the red regions correspond to an excess of baryon asymmetry as compared to the one observed in the Universe, for the maximum value of the phases.

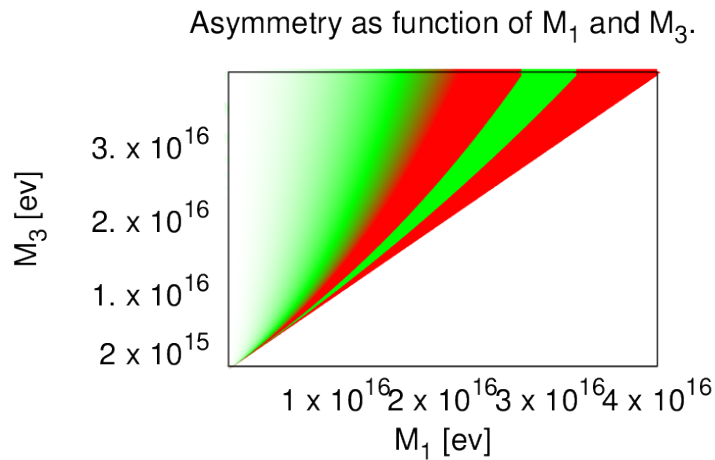


Figure 6: Baryon asymmetry depending on the values of  $M_1, M_3$ . The colour code is similar as in fig. 4.

## 4 Conclusions

The minimal S3-3H extension of the SM accommodates well the masses and mixings of quarks and leptons, and gives naturally a non-zero value for the neutrino reactor mixing angle. We re-derived previous results on the neutrino sector with recent experimental data, taking into account a new value of the phase  $\phi$  to include the angle  $\theta_{12}$  in the model. In the non-degenerate right-handed neutrino mass case, we find a new parametrization of the  $V_{PMNS}$  matrix and use the experimental values in a  $\chi^2$  analysis to fit the new parameters. We find thus a new region in parameter space where the model predicts the mixing angles correctly.

We then calculated the leptogenesis and the associated baryogenesis in this model in the case of two right-handed degenerate neutrino masses, and in the more general case of non-degenerate masses. We show that there are regions in parameter space which allow leptogenesis as a mechanism to solve the observed baryonic asymmetry with right-handed neutrino masses starting from  $10^6$  GeV.

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